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ABSTRACT. Let $\{a_1, a_2, \dots, a_n, \dots\}$ be a sequence of complex numbers which has at most polynomial growth and satisfies an extra assumption. In this talk, inspired by a recent work of Sasane, we give an explanation of the sum

$$a_1 + 2a_2 + 3a_3 + \dots + na_n + \dots,$$

and more generally, for any $k \in \mathbb{N}$, the sum

$$1^k a_1 + 2^k a_2 + 3^k a_3 + \dots + n^k a_n + \dots,$$

from the viewpoint of distributions. As applications, we explain the following summation formulas

$$1^k - 2^k + \dots + (-1)^k = E_k(0)$$

$$\sum_{n=1}^{\infty} \frac{1}{n^k} = \zeta(k) = \frac{(-1)^{k+1} (2\pi)^{-k}}{(k-1)!} \sum_{n=1}^{\infty} n^{k-1} \log n$$

where $\zeta(s)$ is the Riemann zeta function and $E_k(x)$ is the Eulerian polynomial. We also explain the following summation formula

$$\sum_{n=1}^{\infty} \frac{1}{n^k} = \frac{(-1)^{k+1} (2\pi)^{-k}}{(k-1)!} \sum_{n=1}^{\infty} n^{k-1} \log n$$
