

# Workshop on mathematical theory of hydrodynamic stability

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9:00-9:35	On the two-phase free boundary problem for viscous fluid motion	
9:35-10:10	Global existence and decay of solutions to compressible Prandtl system with small analytic data	
10:10-10:40		
10:40-11:15	Tollmien-Schlichting waves in the subsonic regime	
11:15-11:50	Fredholm backstepping for critical operators and application to rapid stabilization for the linearized water waves	
<b>(12:00-14:00)</b>		

## On the two-phase free boundary problem for viscous fluid motion

We consider in this talk the two-phase free boundary problem for viscous fluid motion. This model consists of the incompressible Navier-Stokes equations coupled with the compressible Navier-Stokes equations. We establish the global stability of the two-phase free boundary problem in the absence of surface tension effect with a steady-state violating Rayleigh-Taylor instability and the reference domain being the horizontal infinite layer.

## Global existence and decay of solutions to compressible Prandtl system with small analytic data

In this talk, we shall prove the global existence and the large time decay estimate of solutions to compressible Prandtl system with small initial data, which is analytical in the tangential variable. This is joint work with Yuhui Chen and Minling Li.

## Tollmien-Schlichting Waves near Neutral Stable Curve

In this talk, we consider the linear stability of the boundary layer flows over a flat plate. Tollmien, Schlichting, Lin et al found that there exists a neutral curve, which consists of two branches: lower branch  $\alpha_{\text{low}}(\text{Re})$  and upper branch  $\alpha_{\text{up}}(\text{Re})$ , where  $\alpha$  is the wave number and  $\text{Re}$  is the Reynolds number, such that for any  $\alpha \in (\alpha_{\text{low}}, \alpha_{\text{up}})$ , there exist unstable modes so called Tollmien-Schlichting(T-S) waves to the linearized Navier-Stokes system around the boundary layer flow. These waves play a key role during the early stage of the boundary layer transition. In a breakthrough work Grenier, Guo and Nguyen gave a rigorous construction of the T-S waves. In this work, we confirm the existence of neutral stable curve. To end this, we develop a more delicate method solving the Orr-Sommerfeld equation by borrowing some ideas from the triple-deck theory, which allows to construct the T-S waves in a neighborhood of neutral curve.

## Fredholm backstepping for critical operators and application to rapid stabilization for the linearized water waves

Fredholm-type backstepping transformation, introduced by Coron and Lü, has become a powerful tool for rapid stabilization with fast development over the last decade. Its strength lies in its systematic approach, allowing to deduce rapid stabilization from approximate controllability. But limitations with the current approach exist for operators of the form  $|D_x|^\alpha$  for  $\alpha \in (1, 3/2]$ . We present here a new compactness/duality method which hinges on Fredholm's alternative to overcome the  $\alpha=3/2$  threshold. More precisely, the compactness/duality method allows to prove the existence of a Riesz basis for the backstepping transformation for skew-adjoint operator verifying  $\alpha > 1$ , a key step in the construction of the Fredholm backstepping transformation, where the usual methods only work for  $\alpha > 3/2$ . The illustration of this new method is shown on the rapid stabilization of the linearized capillary-gravity water wave equation exhibiting an operator of critical order  $\alpha=3/2$ . This is a joint work with Amaury Hayat, Ludovick Gagnon and Christophe Zhang.

## Periodic homogenization for $P(\phi)_2$

We consider the periodic homogenization for a stochastic quantization equation  $P(\phi)_2$ , containing two limiting procedures, renormalization in singular stochastic PDEs and homogenization. Our result shows these limiting procedures commute in some suitable settings.

## Localization for quasi-periodic Schrodinger Operators on $Z^d$ with $C^2$ -cosine Like Potentials

Anderson Localization (pure point spectrum with exponentially decaying eigenfunctions) is an important phenomenon in spectrum theory for quasi-periodic (QP) operators. In this talk, we will discuss lattice QP Schrodinger operators on  $\mathbb{Z}^d$  with  $C^2$ -cosine like potentials. We will show quantitative Green's function estimates and the arithmetic version of Anderson localization for such QP Schrödinger operators. This talk is based on a joint work with my advisors Zhifei Zhang (Peking University) and Yunfeng Shi (Sichuan University)