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## 三维不可压缩欧拉方程小截面涡环解的轨道稳定性

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## Eventual continuity approach to verifying unique ergodicity of SPDEs

We formulate a new criterion of the asymptotic stability for some non-equicontinuous Markov semigroups, the so-called eventually continuous semigroups. In particular, we provide a non-equicontinuous Markov semigroup example with essential randomness, which is asymptotically stable. We further apply the eventual continuity approach to the study of the ergodicity of stochastic partial differential equations with multiplicative noise. We apply the generalized coupling method to verify the eventual continuity and combine it with the uniform irreducibility to verify the unique ergodicity.

## Global uniqueness of incompressible inhomogeneous Navier-Stokes equations with bounded density

We consider the uniqueness of 3-D incompressible inhomogeneous Navier-Stokes equations in a critical regularity framework, that is  $u_0 \in \dot{B}^{-1+\frac{3}{p}}_{p,1}$  for  $2 < p < 3$ , in the case where the initial density is close to a positive constant in  $L^\infty$  but has no regularity. This result extends the previous results in [Danchin, Wang 2022], here we need additional  $u_0 \in L^n(\mathbb{R}^3)$  for  $n > 3$  instead of  $u_0 \in L^2(\mathbb{R}^3)$ . We also

give a uniqueness statement in the case of  $u_0 \in \{\dot{B}^{-1+\frac{3}{p}}\}_{p,1} \cap L^2\} + \{\dot{B}^{-1+\frac{3}{p}}\}_{p,1} \cap L^n\}$ .

## Some results on global manifolds for dissipative PDEs

In this talk, I give a brief description of the existence results on finite-dimensional global manifolds for several types of dissipative PDEs, including phase-field systems, FitzHugh-Nagumo systems and non-autonomous hyperdissipative Navier-Stokes equations. These are joint works with Profs. Alain Miranville, Rong-Nian Wang and Jianhong Wu. Finally, some ideas for further research are involved.

## A relative entropy approach to the phase transition problems

In this talk we shall first review the relative entropy method by Fischer--Laux--Simon. One feature of the entropy there is that it controls both discrepancy of the Allen-Cahn energy and the calibration of the level-set of the phase field function. It is also compatible with the gradient flow structure so that a Gronwall inequality can be derived without Modica's maximal principle. In the study of vector-valued phase transition problems, it provides estimates of vectorial nature that lead to effective geometric motions when combined with weak convergence methods.